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A CONSTANT WORK MODEL FOR STUDENT ATTENDANCE AND ENROLLMENT

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ABSTRACT

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ABSTRACT

The authors develop a model of undergraduate student attendance that relies on five parameters, one of these being a parameter of total work, w, required to complete the degree. An enrollment forecasting method consistant with these attendance patterns is developed and compared with data for the period, 1961-1966, and a cohort of 2126 and 3298 freshmen entering in the Fall semesters of 1955 and 1960, respectively.

Under the assumptions of the model, the probability of graduation is shown to be the w-th power of the conditional probability of successful completion of a unit of work given that a student drops out or attends and successfully completes a unit of work.

I. INTRODUCTION

(1) Formulation

In several related papers (Oliver and Marshall (1969); Marshall, Oliver and Suslow, (1969)) the authors attempt to correlate data obtained from analysis of student attendance patterns with enrollment forecasts made over successive time periods. In both of these papers the authors rely heavily on linear models in which transition probabilities are used to estimate the movement of students between grades . Thus, it is possible to make use of results which can be derived from the theory of Markov Chains (Bartholomew (1967); Gani (1963); Feller (1957)).

In these models and the data that supports them there is a very apparent break point in the attendance patterns of students at the end of the 8th semester (4th year) after initial registration. The effect can be seen in Figure (1) which shows the fraction of students having consecutive attendance patterns over a twelve semester period.

The purpose of the present paper is to propose a simple attendance and enrollment model based on "constant work" that is required of students before they can obtain a degree. We show that most of the relevant qualitative characteristics of undergraduate student enrollments and attendance patterns at the Berkeley campus of the University of California can be analysed and predicted with reasonable accuracy by such a model.

The probability model that we propose is simple in several respects: it assumes that, except for a quantitative measure of work completed towards a degree, a student makes a decision to attend, vacation or drop out with probabilities p, q, and r respectively, such that $p+q+r = 1$. These probabilities are independent of time or whether the student is freshman, sophomore,

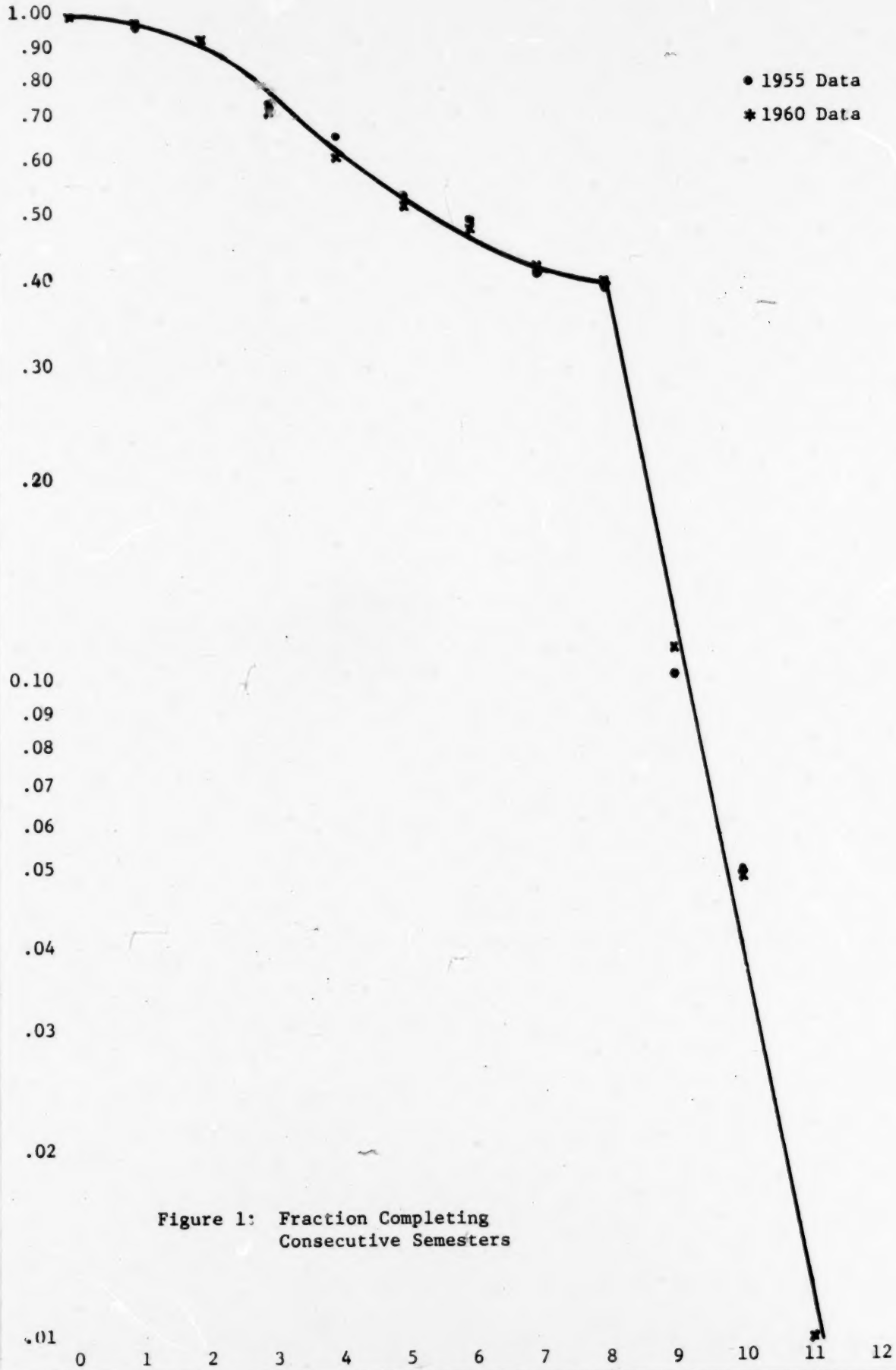

Figure 1: Fraction Completing Consecutive Semesters

junior or senior. If a student attends the university, the conditional probability that he will complete one unit out of a total of w units of work is s; the conditional probability that he fails to complete a unit of work in the semester is $1-s$. The following description may help clarify our use of the words, "attend", "vacation" and "drop-out". An attending student is enrolled in the university. The term, "drop-out" is used to denote a student that permanently leaves the system before graduation, independent of his academic standing. We distinguish our use of this word from the more common usage which has the connotation of poor academic performance. A student on vacation is not a "drop-out" nor is he enrolled; in the next semester he may elect to stay on vacation, attend or drop-out.

(2) Notation

Besides the four probabilities mentioned above and the constant work parameter w, we will denote the random number of consecutive attendances by N_c, and the random number of semesters that elapse between initial registration and graduation or drop out by N. Furthermore, A_n is the event: attend the n^{th} semester after initial registration; B_n is the event: vacation during the n^{th} semester after initial registration, and C_n is the event: graduate or drop out from the university on or before n^{th} semester. Since these events are mutually exclusive and exhaustive,

$$\Pr\{A_n\} + \Pr\{B_n\} + \Pr\{C_n\} = 1. \tag{1}$$

In our sample spaces, graduation or drop out is guaranteed; thus,

$$\lim_{n\to\infty} \Pr\{C_n\} = 1. \tag{2}$$

The relevant distributions of attendance, consecutive attendance, drop out and graduation, as well as the linear equations for expected future enrollments, are derived in Sections II and III. Our paper concludes by comparing some theoretical results with statistical data obtained from an examination of a cohort of 3298 freshmen enrolled for the first time in the Fall semester of 1960, and 2126 freshmen from 1955, at the Berkeley campus of the University of California.

II. ATTENDANCE PATTERNS

(1) Consecutive Attendances

The second through fifth columns of Table 1 show the number and fraction of those students that entered as freshmen in the Fall semesters of 1955 and 1960 and completed each consecutive semester with no interruptions in attendance (Suslow, 1968). These fractions are plotted in Figure (1). More will be said about this graph and other statistical data on student attendance patterns in companion papers but, we now wish to point out and investigate the very apparent break point at the eighth semester. We believe that this characteristic structure may be the result of students having a "constant work" requirement. Such a concept is inherent in certain Markov models that we discuss in other papers but our interest here is to develop a simple independent trials model that clearly illustrates this basic point. It is also interesting to note that the 1955 and 1960 data indicate stationary student attendance patterns.

Number of Consecutive Semesters Completed	1955 Entering Freshmen (2,126 students)		1960 Entering Freshmen (3,298 students)	
	Number of Students	Per cent	Number of Students	Per cent
0	2,126	100	3,298	100
1	2,067	97	3,228	98
2	1,923	90	2,994	91
3	1,554	73	2,301	70
4	1,373	65	2,018	61
5	1,112	52	1,679	51
6	1,027	48	1,554	47
7	883	41	1,371	42
8	819	38	1,291	39
9	222	10	363	11
10	112	5	181	5
11	15	1	33	1

Table 1: Students Completing Each Consecutive Semester With No Interruptions in Attendance

We assume that as long as a student has not completed w units of work, the probability of attending a semester is p. Let N_c be the number of consecutive attendances of a student after he first enters. For the first w semesters, any break in the attendance pattern can be caused only by a student vacationing or dropping out. Hence,

$$\Pr\{N_c \geq n\} = p^n \qquad n = 0,1,2, \ldots, w. \tag{3}$$

For a student to attend at least $w+k$ semesters without interruption, he must attend all $w+k$ of them and successfully complete no more than $(w-1)$ in the first $(w+k-1)$ semesters. Hence we see that the probability,

$$\Pr\{N_c \geq w+k\} = p^{w+k} \sum_{j=0}^{w-1} \binom{w+k-1}{j} s^j (1-s)^{w+k-1-j}, \quad k = 1,2, \ldots. \tag{4}$$

is the product of a geometric and incomplete binomial term. We can rewrite this expression

$$\begin{aligned} \Pr\{N_c \geq n\} &= p^n && n = 0,1,2, \ldots w \\ &= p^n\left[1-\sum_{j=w}^{n-1}\binom{n-1}{j} s^j (1-s)^{n-1-j}\right] && n = w+1, w+2, \ldots. \end{aligned} \tag{5}$$

Perhaps the most revealing way to show the effect of the w semesters of work is to ask for the conditional probability that $n+1$ consecutive semesters will be attended given that n have already been attended:

$$\begin{aligned} \Pr\{N_c \geq n+1 \mid N_c \geq n\} &= p && 1 \leq n \leq w \\ &= p(1-s^w) && n = w \\ &= p\frac{1-s^w\ (1 + w(1-s))}{1-s^w} && n = w+1\ . \end{aligned} \tag{6}$$

A calculation of these conditional probabilities for the first ten terms

when $w = 8$, $p = .89$, $s = .953$ yields the sequence, .89, .89, .89, .89, .89, .89, .89, 0.32, 0.18 and indicates how rapidly the conditional probability for the first nine terms of continuous attendance decreases after the eighth semester. In the first eight semesters, the fraction of continuous attendance students decreases geometrically but, beyond eight, the drop is much more rapid. The curve exhibited by the real data in the 1 to 8 semester range is due to the dependencies between states which are ignored in this simple model and is the subject of more detailed analysis in the two papers to which we have already referred.

Semester	1955 data	1960 data	$\Pr[N_c \geq n]^*$
0	1.00	1.00	1.00
1	.97	.98	.89
2	.90	.91	.79
3	.73	.70	.70
4	.65	.61	.63
5	.52	.51	.56
6	.48	,47	.49
7	.41	.42	.44
8	.38	.39	.39
9	.10	.11	.11
10	.05	.05	.02
11	.01	.01	.00

*$p = 0.890$ $s = .953$

Table 2: The Comparison of Equation (5) with 1955 and 1960 Cohort Data.

(2) Attending and Vacationing Students

Attendance in any one of the first w semesters simply requires that (a) the student attend and (b) he not drop out prior to that semester. Thus:

$$\Pr\{A_1\} = p$$

$$\Pr\{A_2\} = (1-r)p$$

$$\Pr\{A_w\} = (1-r)^{w-1}p \quad . \tag{7}$$

Attendance in the n^{th} $(n \geq w)$ semester requires that no more than $w-1$ units of work be completed up to that point and, furthermore, that the student not drop out. Thus:

$$\Pr\{A_n\} = p\sum_{j=0}^{w-1}\binom{n-1}{j}(ps)^j\left(q+p(1-s)\right)^{n-1-j} \qquad n=w,\ w+1,\ w+2,\ \ldots\ldots, \tag{8}$$

The first p on the right hand side ensures the students' presence on the n^{th} semester. Factoring out the term $(1-r)^{n-1}$ from the right hand side of Equation (8) yields the expression:

$$\Pr\{A_n\} = p(1-r)^{n-1}\left[\sum_{j=0}^{w-1}\binom{n-1}{j}x^j(1-x)^j\right] \qquad \begin{matrix} n \geq w, \\ x = \frac{ps}{1-r} \end{matrix} \tag{9}$$

which, of course, reduces to Equation (7) when $n=w$. The factor within square brackets is an incomplete binomial sum which is less than one; thus, the probability of attendance decreases more rapidly for $n \geq w$ than it does for $n \leq w$.

The probability of being on vacation on semester n is simply obtained by substituting q for p in Equations (7) and (9),

$$\begin{aligned} \Pr\{B_n\} &= q(1-r)^{n-1} && 1 \leq n \leq w \\ &= q(1-r)^n \sum_{j=0}^{w-1}\binom{n-1}{j}\left(\frac{ps}{1-r}\right)^j\left(\frac{q+p(1-s)}{1-r}\right)^{n-1-j} && w \leq n \ . \end{aligned} \tag{10}$$

(3) Fraction Graduating and Total Elapsed Time

The fraction graduating from a freshman class is easy to obtain from the distribution of waiting time (in semesters) until graduation. Clearly, an entering freshman can not graduate in anything less than w semesters if he advances at the maximum rate of one unit of work per semester. Thus, if N_g is the random number of semesters to graduation,

$$\Pr\{N_g = n\} = 0 \qquad n = 1,2,3, \ldots, w-1 . \quad (11)$$

To graduate at the end of the n^{th} semester $(n \geq w)$, the requirement is simply that in $n-1$ previous semesters, $w-1$ units of work are completed successfully; in the remaining $n-w$ semesters, the student either does not attend or, if he attends, he fails to complete a unit of work; finally, he must successfully complete the last semester. Except for the last semester, the exact timing of failures or vacations is immaterial to the completion of work. Thus, the waiting time to graduation has the negative binomial distribution:

$$\Pr\{N_g = n\} = ps\left[\binom{n-1}{w-1}(ps)^{w-1}(q+p(1-s))^{n-w}\right] \qquad n=w, w+1, \ldots . \quad (12)$$

By adding the terms in Equation (12) for all $n \geq w$, we obtain the probability of ever graduating. To obtain this sum we make use of the identity:

$$\sum_{k=0}^{\infty}\binom{w+k-1}{k}x^w(1-x)^k = 1 \quad \text{if} \quad 0 \leq x \leq 1, \quad (13)$$

and obtain the result,

$$\sum_{n=w}^{\infty}\Pr\{N_g=n\} = \sum_{k=0}^{\infty}\binom{w+k-1}{k}(ps)^w(q+p(1-s))^k$$

$$= \left(\frac{ps}{r+ps}\right)^w \sum_{k=0}^{\infty}\binom{w+k-1}{k}(r+ps)^w(1-r-ps)^k \quad .$$

Thus, the probability of ever graduating is

$$\Pr\{\text{graduation}\} = \left(\frac{ps}{r+ps}\right)^{w} \quad . \tag{14}$$

The probability of graduating or dropping out on or before semester n is obtained by noting that for $n \leq w-1$ no one graduates, and

$$\Pr\{C_n\} = \sum_{j=1}^{n} r(1-r)^{j-1} = 1-(1-r)^{n} \quad . \qquad 1 \leq n \leq w-1 \quad . \tag{15}$$

This probability is one minus the tail distribution that drop-out occurs after the n^{th} semester. The probability distribution for $n \geq w$ is only slightly more complicated. If N is the random variable measuring the total number of semesters that elapse between registration and graduation or drop-out, we make use of the cumulative distribution of N ,

$$\sum_{j=1}^{n} \Pr\{N=j\} = \Pr\{C_n\} \tag{16}$$

and Equations (12) and (8) to obtain

$$\Pr\{N=n\} = ps\left[\binom{n-1}{w-1}(ps)^{w-1}\left(q+p(1-s)\right)^{n-w}\right] + r\left[\sum_{j=0}^{w-1}\binom{n-1}{j}(ps)^{j}\left(q+p(1-s)\right)^{n-1-j}\right].$$

By factoring $(1-r)^{n-1}$ terms and summing the incomplete binomial expressions:

$$\Pr\{C_n\} = 1-(1-r)^{n}\left[1-\sum_{j=w}^{n-1}\binom{n-1}{j}(x)^{j}(1-s)^{n-1-j}\right] \qquad w \geq n; \quad x=\frac{ps}{1-r} \quad . \tag{17}$$

Thus, the requirement of Equation (1) is satisfied.

(4) Correlation with Cohort Data

The purpose of this section is twofold: (i) to show how parameters estimated from certain statistics of the freshman cohort population can be used to make theoretical computations of the distributions that we have already derived in previous sections, and (ii) to show how these theoretical computations agree with independently obtained data on the attendance patterns of undergraduate students. The reader should bear in mind that we have not attempted to take into account the more realistic and difficult interdependencies which arise as students proceed through grades; their attendance patterns are affected by performance to date, change in majors, curricula and so on; none of these important effects are explicitly calculated.

The manner in which we estimate the parameters p, q, r and s is straight forward. Since the distribution of consecutive attendances does not explicitly involve the parameters r or q, we used the slope of this curve (Figure 1) before and after $w=8$ to estimate p and s. From Table 1 and Equation (5) we see that

$$\Pr\{N_c \geq 8\} = p^8 = 0.390 \quad .$$

At $n=9$, we require that

$$p^9(1-s^8) = 0.111$$

From these equations we estimate that $p = .890$ and $s = .953$.

Using these values of p and s in equation (14), we find the probability of graduating as a function of r, and tabulate these probabilities in Table 3. Notice that the probability of graduation is highly sensitive to the drop-out probability r even when ps is close to 1. A change in r of 2 per cent from .06 to .08 leads to a 9 per cent drop in

graduation.

r	$\left(\frac{ps}{r+ps}\right)^8$
0.00	1.000
0.02	0.830
0.04	0.692
0.06	0.579
0.08	0.486
0.10	0.410

Table 3: Probability of Graduation as a Function of the Drop-out Probability. (p=.890, s=.953)

Berkeley campus records (Suslow, et al (1968))indicate that 52 per cent of the entering freshmen in the 1955 and 1960 group eventually graduate. To estimate r, we therefore require that

$$\left(\frac{ps}{r+ps}\right)^8 = 0.52 \quad .$$

Since $(ps) = .848$, this equation gives $r = .072$. Also, since $p+q+r = 1$, we have $q = .038$, $q+p(1-s) = .080$. Using these values, we calculate $Pr\{A_n\}$ in Figure (2). The solid line is the theoretical curve predicted by Equations (7) and (9); the 1955 and 1960 cohorts are represented by the data points. Notice that attendance of students in the early years is underestimated while attendance in the fourth through seventh semesters is overestimated. However, the rapid decrease in the probability of attendance in the ninth, tenth and eleventh semesters seems to agree well with the data.

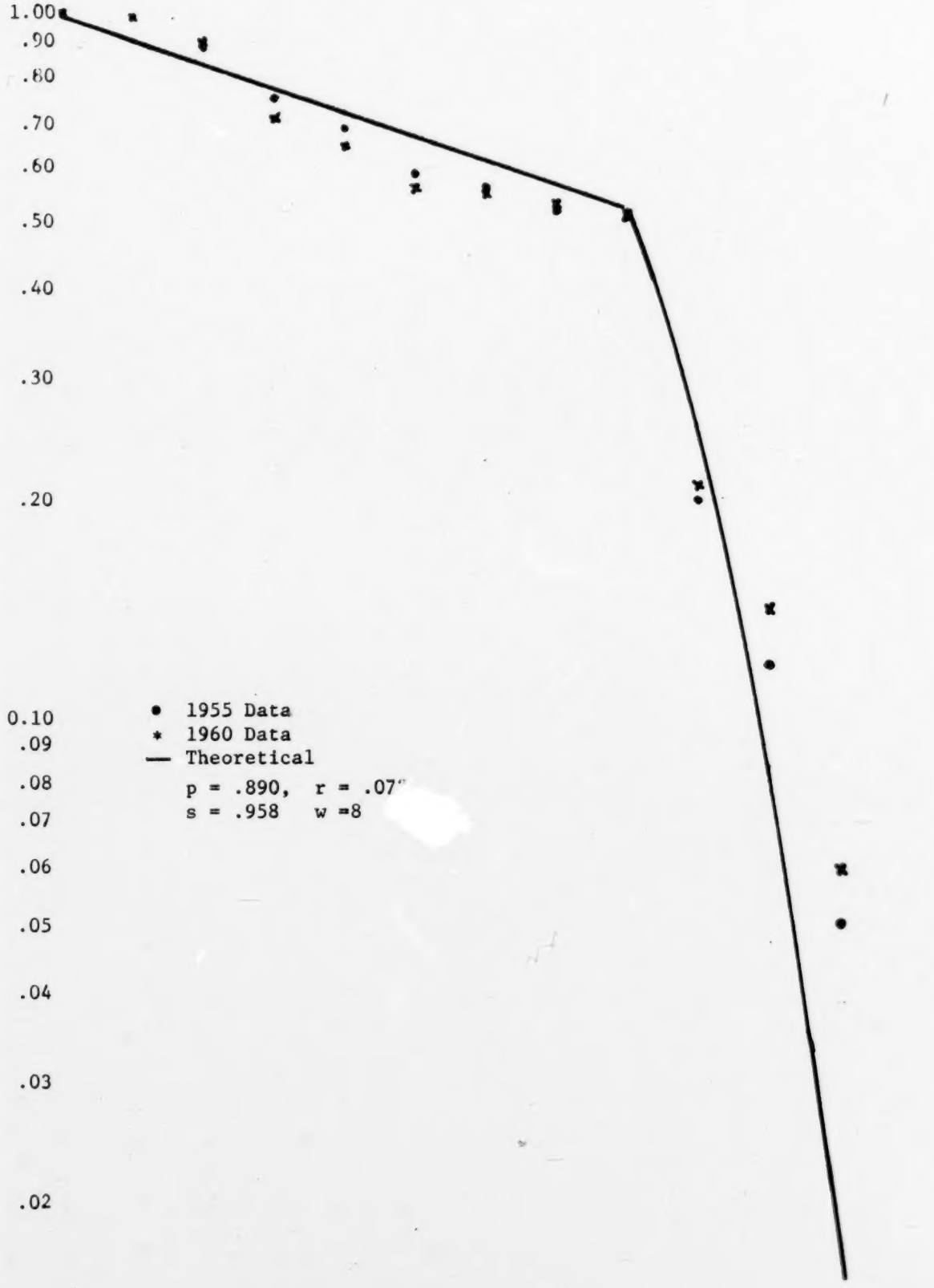

Figure 2: The Probability of Attendance

III. ENROLLMENT FORECASTS

(1) The Forecasting Model

Denote the expected number of students at semester t that have completed j units of work by $x_j(t)$. By this definition, enrollments include both vacationing and attending students. If a student attends and successfully completes a semester (with probability ps) , he moves to a group with one more unit of work completed. If he attends and fails, or goes on vacation (with probability $q+p(1-s)$), his level of work remains unchanged. Of course, if he drops out or graduates in a semester, he is not counted in the next. Let $y_j(t)$ be the new admissions at level j into semester t . Thus, the equations of motion for enrollments in semester $t+1$ in terms of enrollments in semester t are

$$x_j(t+1) = psx_{j-1}(t) + (q+p(1-s))x_j(t) + y_j(t+1) \qquad (18)$$

$$j = 1,2, \ldots, w-1 ,$$

$$t = 0,1,2, \ldots .$$

with the special case,

$$x_0(t+1) = (q+p(1-s))x_0(t) + y_0(t+1),$$

for those students who do not complete any units of work. Equation (18) can be solved recursively for $x_0(t+1)$ in terms of the sequences:

$$(y_j(1)), ((y_j(2)), \ldots\ldots (y_j(t+1)) \qquad 0 \leq j \leq w-1 \qquad (19)$$

and, given initial conditions,

$$x_1(0), x_2(0), \ldots\ldots x_{w-1}(0). \qquad (20)$$

With $X(t) = (x_j(t))$ and $Y(t) = (y_j(t))$ representing the row vectors of expected student enrollments and admissions, and P representing the transition probabilities, Equation (18) can be written in matrix form as

$$X(t+1) = X(t)P + Y(t+1). \tag{21}$$

Again, $X(0) = (x_j(0))$ is given. It is well known (Feller, (1968); Bartholomew, (1967)) that as $Y(t)$ approaches a limiting vector Y with constant growth rate, the limiting enrollment vector can be written:

$$X = Y(I-P)^{-1} \quad . \tag{22}$$

The inverse of $(I-P)$ exists because all rows except the last sum to $1-r<1$ and elements in the last row corresponding to $j=w-1$ sum to $1-r-ps<1$. The elements of the j^{th} column of $(I-P)^{-1}$ can be explicitly obtained in powers of $\left(\frac{ps}{r+ps}\right)$; thus one can write the expected number of students with j units of work as:

$$x_j = (r+ps)^{-1} \sum_{i=0}^{j} y_{j-i}\left(\frac{ps}{r+ps}\right)^i \quad . \tag{23}$$

This steady-state formulation of the enrollment model is particularly interesting because we are now in a position to show that characteristics of students predicted by Equation (23) agree with theoretical results we have already derived in Section II for attendance patterns of entering students. Thus, an experimental and theoretical link between student attendance and enrollment patterns can be clearly established.

To illustrate the consistency of these results, we consider the

special solution of Equation (23) when $y_0 = 1$ and $y_j = 0$, $j \neq 0$. The expected number that graduate and drop out in the t^{th} semester are given respectively by

$$psx_{w-1}(t) \quad \text{and} \quad r\sum_{j=0}^{w-1} x_j(t). \tag{24}$$

Substituting Equation (23) into (24) yields the result:

$$\Pr\{\text{graduation}\} = ps(r+ps)^{-1} y_0\left(\frac{ps}{r+ps}\right)^{w-1} = \left(\frac{ps}{r+ps}\right)^{w} \tag{25}$$

which agrees with Equation (14)).

Furthermore,

$$\Pr\{\text{drop-out}\} = r\sum_{j=0}^{w-1} (r+ps)^{-1} y_0\left(\frac{ps}{r+ps}\right)^{j} \quad . \tag{26}$$

By making use of the formula,

$$1+a+a^2+\ldots+a^{w-1} = \frac{1-a^w}{1-a} \quad , \tag{27}$$

we obtain

$$\begin{aligned}\Pr\{\text{drop-out}\} &= r(r+ps)^{-1} \left[\frac{1-(ps/r+ps)^w}{1-(ps/r+ps)}\right] \\ &= 1 - \left(\frac{ps}{r+ps}\right)^w \\ &= 1 - \Pr\{\text{graduation}\} \quad .\end{aligned} \tag{28}$$

(2) Forecasts of Lower and Upper Division Students in 1961-1966

As an independent experimental confirmation of the basic structure of our constant work model, we made forecasts of returning lower division (freshmen plus sophomores and upper division students (juniors plus seniors) in the five-year period, 1961-1966. Admission data for new students is given in Table (4). Recall that the forecasting problem associated with Equation (18) is one of predicting vectors $X(t)$ for a given sequence of $Y(t)$ vectors. No attempt has been made in this paper to develop or test models forecasting new admissions per se.

The choice of $X(0)$, initial enrollments, is difficult since data on new admissions, attending, vacationing or returning students has never been collected or classified in the units of work natural to our model.. Somewhat arbitrarily we chose to assign the work indices

$$\begin{aligned} j &= 0,1 \quad \text{to freshmen} \\ &= 2,3 \quad \text{to sophomores} \\ &= 4,5 \quad \text{to juniors} \qquad\qquad w=8 \\ &= 6,7 \quad \text{to seniors} \end{aligned}$$

and selected the fractions

$$\frac{x_0}{x_0+x_1} = 0.890 \qquad \frac{x_2}{x_2+x_3} = .785$$

$$\frac{x_4}{x_4+x_5} = .768 \qquad \frac{x_6}{x_6+x_7} = .714 \tag{29}$$

to indicate the split of freshmen, sophomores, juniors and seniors into work levels for the year 1961.

A good allocation of new admission data in Table (4) into the appropriate work levels is also not obvious; we chose to allocate all fall admissions

into the even numbered work levels, and all spring admissions into the odd numbered levels based on our intuitive feeling that spring entrants would wait for a fall admission date if their completed work level were such that they could be in phase with the normal course curricula offered at that time.

Our model assumes that new admissions and returning students are counted at one point in time whereas, in reality, approximately fifteen weeks elapse between the beginning and end of a semester; attrition of returning students, drop-outs and new admissions leave and enter the system at many non-overlapping periods of time. To simplify the structure of our accounting system, we assumed that *all* new admissions to semester t attend that semester. Hence, vacationing students can only be drawn from the population of students that have been enrolled in at least one prior semester. We know that in real life, many new admissions leave before the semester is completed; it is difficult to include such effects at this time because none of our University records have ever considered a separate category of vacationing students.

From Equations (18) and (21) we see that the expected number on vacation at the t^{th} period is

$$\frac{q}{q+p}\left[X(t-1)P\right] = \frac{q}{q+p}\left[X(t) - Y(t)\right] . \tag{30}$$

Subtracting this number from X(t) yields the expected number attending at the t^{th} semester:

$$\frac{p}{q+p} X(t) + \frac{q}{q+p} Y(t). \tag{31}$$

Using the new admissions of Table (4), we obtained initial values,

	Spr '62	Fall '62	Spr '63	Fall '63	Spr '64	Fall '64	Spr '65	Fall '65	Spr '66	Fall '66
Freshmen	324	3,528	328	3,632	346	3,443	256	2,590	291	3,072
Sophomores	204	678	187	732	209	609	180	396	210	742
Juniors	303	1,416	324	1,568	173	1,443	452	1,035	476	1,406
Seniors	30	184	42	196	45	202	49	126	66	205

Table 4: New Admissions on Berkeley Campus, Spring 1962 - Fall 1966.

X(0) = X(1961), by requiring that the number of attending students calculated from Equation (31) agree with the registrar records for students enrolled in the fall of 1961. These figures are shown in the first column of Table 5. Forecasts of students at each level were obtained from Equation (21) and expected numbers attending were then calculated from (31). The results of these calculations are recorded in the top row of each division; actual historical figures are recorded beneath them.

	1961		1962	1963	1964	1965	1966
Lower Division	7621	Forecasts	8120	8491	8316	7023	7094
		Actual	7621	7952	7654	6656	6759
Upper Division	8123	Forecasts	8807	9781	10183	10082	9990
		Actual	8972	9595	10014	9892	9988
Total	15744	Forecasts	16927	18272	18499	17105	17084
		Actual	16,593	17547	17668	16548	16747

Table 5: Lower and Upper Division: Forecasts and Actual Enrollments (1961-1966)

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